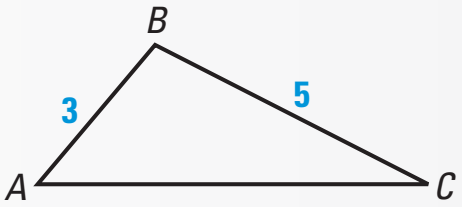
Inequalities in one triangle

- I can use the inequalities in one triangle to solve problems.
- I can state and apply the triangle inequality theorem.

THEOREMS

THEOREM 5.10

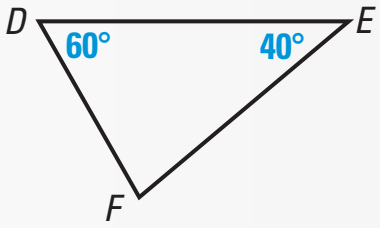
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$m\angle A > m\angle C$

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



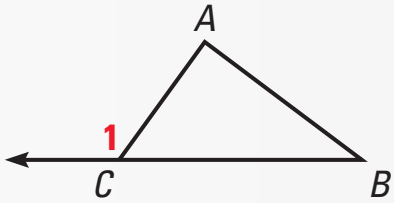
$EF > DF$

THEOREM

THEOREM 5.12 *Exterior Angle Inequality*

The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.

$m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$



THEOREM

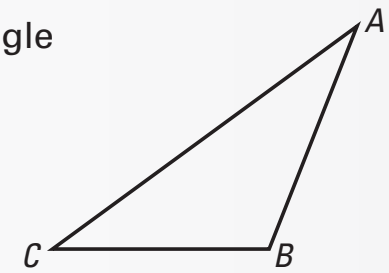
THEOREM 5.13 *Triangle Inequality*

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$

$AC + BC > AB$

$AB + AC > BC$



EXAMPLE 4 *Finding Possible Side Lengths*

A triangle has one side of 10 centimeters and another of 14 centimeters. Describe the possible lengths of the third side.

SOLUTION

Let x represent the length of the third side. Using the Triangle Inequality, you can write and solve inequalities.

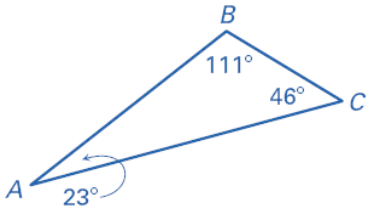
$x + 10 > 14$ $\qquad$ $10 + 14 > x$

$x > 4$ $\qquad$ $24 > x$

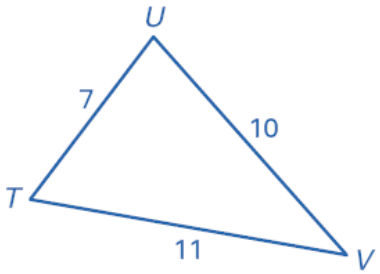
▶ So, the length of the third side must be greater than 4 centimeters and less than 24 centimeters.

Write the measures in the triangles in order from least to greatest.

6.



7.

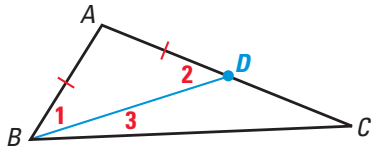


Proof

Theorem 5.11 will be proved in Lesson 5.6, using a technique called *indirect proof*. Theorem 5.10 can be proved using the diagram shown below.

GIVEN ▶ $AC > AB$

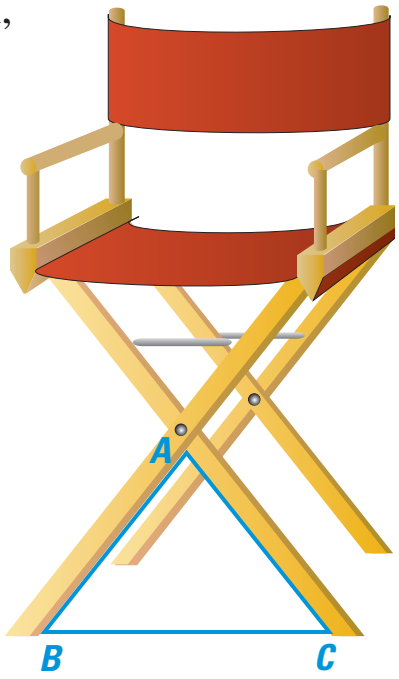
PROVE ▶ $m\angle ABC > m\angle C$



Paragraph Proof Use the Ruler Postulate to locate a point D on $\overline{AC}$ such that $DA = BA$. Then draw the segment $\overline{BD}$. In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$. Because $m\angle ABC = m\angle 1 + m\angle 3$, it follows that $m\angle ABC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle ABC > m\angle 2$. Because $m\angle 2 = m\angle 3 + m\angle C$, $m\angle 2 > m\angle C$. Finally, because $m\angle ABC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle ABC > m\angle C$.

EXAMPLE 2 *Using Theorem 5.10*

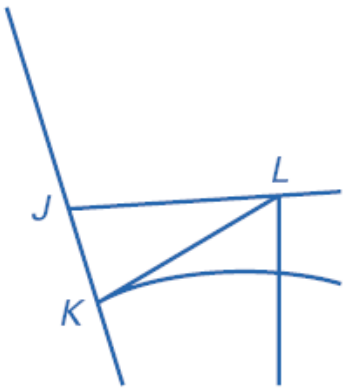
DIRECTOR'S CHAIR In the director's chair shown, $\overline{AB} \cong \overline{AC}$ and $BC > AB$. What can you conclude about the angles in $\triangle ABC$?



SOLUTION

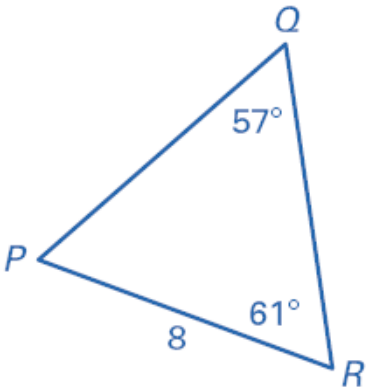
Because $\overline{AB} \cong \overline{AC}$, $\triangle ABC$ is isosceles, so $\angle B \cong \angle C$. Therefore, $m\angle B = m\angle C$. Because $BC > AB$, $m\angle A > m\angle C$ by Theorem 5.10. By substitution, $m\angle A > m\angle B$. In addition, you can conclude that $m\angle A > 60°$, $m\angle B < 60°$, and $m\angle C < 60°$.

8. The figure shows the side view of an Adirondack chair. In the figure, $\overline{JL} \cong \overline{LK}$ & $JK < JL$. What can you conclude about the angles in



9. Name the smallest and largest angle angles of $\triangle PQR$.

10. Is $QR \geq 8$? Is PQ < 8?



EXAMPLE 4 *Finding Possible Side Lengths*

A triangle has one side of 10 centimeters and another of 14 centimeters. Describe the possible lengths of the third side.

SOLUTION

Let x represent the length of the third side. Using the Triangle Inequality, you can write and solve inequalities.

$x + 10 > 14$ $\qquad$ $10 + 14 > x$

$x > 4$ $\qquad$ $24 > x$

So, the length of the third side must be greater than 4 centimeters and less than 24 centimeters.

Is it possible to construct a triangle a with the following side lengths?

11. 3 in, 3 in, 8 in

12. 6 in, 6 in, 12 in

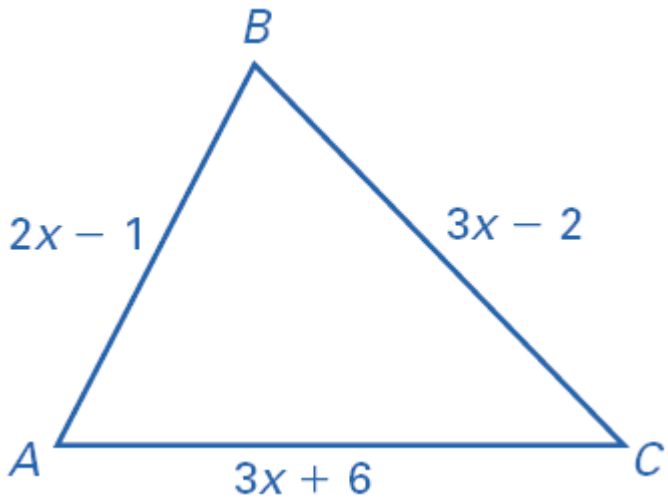
13. 9 in, 5 in, 11 in

Inequalities in one triangle

14. A triangle has one side of 8 cm and another of 17 cm. Describe the possible lengths of the third side.

15. A triangle has one side of 11 in and another side of 16 in. What are the possible lengths of the third side?

16. _____ Use the diagram to solve the inequality AB + BC > AC.



A. $x > \frac{9}{8}$

B. $x < \frac{9}{2}$

C. $x < \frac{7}{2}$

D. $x > \frac{9}{2}$

E. $x > \frac{7}{2}$

17. What angles of a triangle are used in the Exterior Angle inequality?

18. In $\triangle ABC, \measuredangle B$ is obtuse and $m\measuredangle A < m\measuredangle C.$ State what you know about the lengths of the sides of $\triangle ABC.$